OCR
Oxford Cambridge and RSA

Wednesday 14 October 2020 – Afternoon

AS Level Mathematics A

H230/02 Pure Mathematics and Mechanics

Time allowed: 1 hour 30 minutes

8232206127

You must have:
- the Printed Answer Booklet
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total number of marks for this paper is **75**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

ADVICE
- Read each question carefully before you start your answer.

Formulae
AS Level Mathematics A (H230)

Binomial series

$$(a+b)^n = a^n + {}^n\mathrm{C}_1 a^{n-1}b + {}^n\mathrm{C}_2 a^{n-2}b^2 + \ldots + {}^n\mathrm{C}_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$$

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

Differentiation from first principles

$$\mathrm{f}'(x) = \lim_{h \to 0} \frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$$

Standard deviation

$$\sqrt{\frac{\Sigma(x-\overline{x})^2}{n}} = \sqrt{\frac{\Sigma x^2}{n} - \overline{x}^2} \quad \text{or} \quad \sqrt{\frac{\Sigma f(x-\overline{x})^2}{\Sigma f}} = \sqrt{\frac{\Sigma f x^2}{\Sigma f} - \overline{x}^2}$$

The binomial distribution

If $X \sim \mathrm{B}(n, p)$ then $\mathrm{P}(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$, mean of X is np, variance of X is $np(1-p)$

Kinematics

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u+v)t$

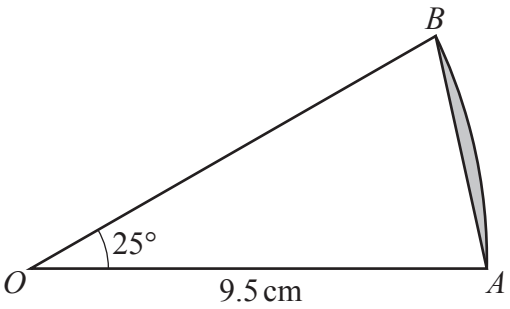
$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

Section A: Pure Mathematics

Answer **all** the questions.

1



The diagram shows a sector AOB of a circle with centre O and radius 9.5 cm. The angle AOB is 25°.

(a) Calculate the length of the straight line AB. **[2]**

(b) Find the area of the segment shaded in the diagram. **[3]**

2 Two curves have equations $y = \ln x$ and $y = \frac{k}{x}$, where k is a positive constant.

(a) Sketch the curves on a **single** diagram. **[3]**

(b) Explain how your diagram shows that the equation $x \ln x - k = 0$ has exactly one real root. **[2]**

3 In this question you must show detailed reasoning.

Find the equation of the normal to the curve $y = 4\sqrt{x} - 3x + 1$ at the point on the curve where $x = 4$. Give your answer in the form $ax + by + c = 0$, where a, b and c are integers. **[7]**

4 **In this question you must show detailed reasoning.**

The cubic polynomial $6x^3 + kx^2 + 57x - 20$ is denoted by $f(x)$. It is given that $(2x - 1)$ is a factor of $f(x)$.

(a) Use the factor theorem to show that $k = -37$. **[2]**

(b) Using this value of k, factorise $f(x)$ completely. **[3]**

(c) **(i)** Hence find the three values of t satisfying the equation $6e^{-3t} - 37e^{-2t} + 57e^{-t} - 20 = 0$. **[2]**

(ii) Express the sum of the three values found in part **(c)(i)** as a single logarithm. **[2]**

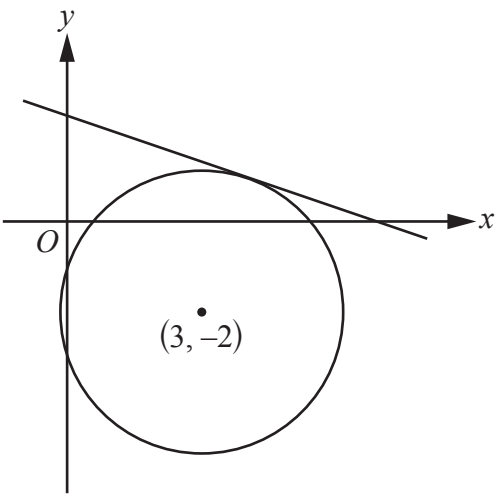
5 A curve has equation $y = a(x + b)^2 + c$, where a, b and c are constants. The curve has a stationary point at $(-3, 2)$.

(a) State the values of b and c. **[2]**

When the curve is translated by $\begin{pmatrix} 4 \\ 0 \end{pmatrix}$ the transformed curve passes through the point $(3, -18)$.

(b) Determine the value of a. **[3]**

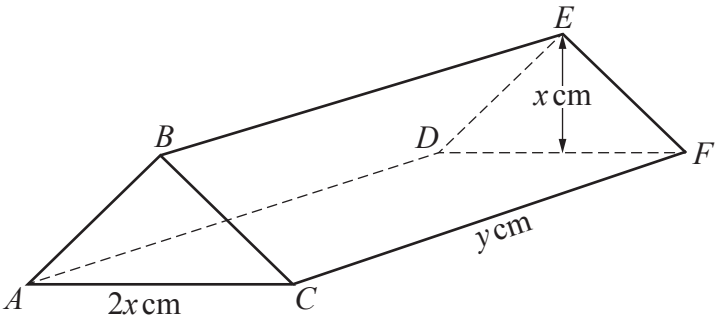
6 **In this question you must show detailed reasoning.**



The diagram shows the line $3y + x = 7$ which is a tangent to a circle with centre $(3, -2)$.

Find an equation for the circle. **[6]**

7



The diagram shows a model for the roof of a toy building. The roof is in the form of a solid triangular prism $ABCDEF$. The base $ACFD$ of the roof is a horizontal rectangle, and the cross-section ABC of the roof is an isosceles triangle with $AB = BC$.

The lengths of AC and CF are $2x$ cm and y cm respectively, and the height of BE above the base of the roof is x cm.

The total surface area of the **five** faces of the roof is $600\,\text{cm}^2$ and the volume of the roof is $V\,\text{cm}^3$.

(a) Show that $V = kx(300 - x^2)$, where $k = \sqrt{a} + b$ and a and b are integers to be determined. **[6]**

(b) Use differentiation to determine the value of x for which the volume of the roof is a maximum. **[4]**

(c) Find the maximum volume of the roof. Give your answer in cm^3, correct to the nearest integer. **[1]**

(d) Explain why, for this roof, x must be less than a certain value, which you should state. **[2]**

Section B: Mechanics

Answer **all** the questions.

8 A particle is in equilibrium under the action of the following three forces:

$(2p\mathbf{i} - 4\mathbf{j})$ N, $(-3q\mathbf{i} + 5p\mathbf{j})$ N and $(-13\mathbf{i} - 6\mathbf{j})$ N.

Find the values of p and q. **[3]**

9 A crane lifts a car vertically. The car is inside a crate which is raised by the crane by means of a strong cable. The cable can withstand a maximum tension of 9500 N without breaking. The crate has a mass of 55 kg and the car has a mass of 830 kg.

(a) Find the maximum acceleration with which the crate and car can be raised. **[2]**

(b) Show on a clearly labelled diagram the forces acting on the **crate** while it is in motion. **[1]**

(c) Determine the magnitude of the reaction force between the crate and the car when they are ascending with maximum acceleration. **[3]**

10 A particle P is moving in a straight line. At time t seconds P has velocity $v\,\mathrm{m\,s^{-1}}$ where $v = (2t+1)(3-t)$.

(a) Find the deceleration of P when $t = 4$. **[2]**

(b) State the positive value of t for which P is instantaneously at rest. **[1]**

(c) Find the total distance that P travels between times $t = 0$ and $t = 4$. **[3]**

11 A car starts from rest at a set of traffic lights and moves along a straight road with constant acceleration $4\,\text{m}\,\text{s}^{-2}$. A motorcycle, travelling parallel to the car with constant speed $16\,\text{m}\,\text{s}^{-1}$, passes the same traffic lights exactly 1.5 seconds after the car starts to move. The time after the car starts to move is denoted by t seconds.

(a) Determine the two values of t at which the car and motorcycle are the same distance from the traffic lights. **[6]**

These two values of t are denoted by t_1 and t_2, where $t_1 < t_2$.

(b) Describe the relative positions of the car and the motorcycle when $t_1 < t < t_2$. **[1]**

(c) Determine the maximum distance between the car and the motorcycle when $t_1 < t < t_2$. **[3]**

END OF QUESTION PAPER